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IMPULSIVE LOADING OF RECTANGULAR PLATES WITH FINITE PLASTIC DEFORMATIONS

by

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and
T. Ariman

November, 1971

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FOREWORD

This technical report was prepared by the Dynamical Systems Group under Project THEMIS at the University of Notre Dame, College of Engineering. The co-authors, Drs. B. Sureshwara, Dr. T. Ariman, and Dr. L. H. N. Lee are respectively Visiting Assistant Professor of Civil Engineering, Valparaiso University, Valparaiso, Indiana and a former graduate research assistant in the Program of Engineering Science, Associate Professor, and Professor in the Department of Aerospace and Mechanical Engineering.

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ABSTRACT

This paper is concerned with the responses of inelastic rectangular plates to impulsive loadings. The effects of strain hardening and strain rate sensitivity of the material are taken into consideration in this analysis. A variational principle in dynamics of inelastic bodies subject to finite deformation is used to determine the deformation process of the plate. A sandwich plate idealization is employed. The accuracy of the numerical solution is evaluated by comparing it with the existing analytical and experimental results. The results indicate that this method is adequate for determining the dynamic behavior of inelastic rectangular plates at relatively large deformations.

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INTRODUCTION

The general problem of inelastic deformations in structures subjected to dynamic loading has received wide interest in recent years. There is need to correctly estimate the damage inflicted as a result of blast loadings, earthquakes, etc., on structures. These impulsive forces cause damage to the structures in the form of large permanent deformations which usually involve the effects of both geometric and material constitutive non-linearities.

In most of the previous studies, the concepts of limit analysis and rigid-plastic idealization have been applied to obtain closed form solutions for permanent plastic deformations of beams, plates, etc. [1-8]. However, the rigid-plastic idealization is applicable only to problems in which the elastic energy of a structure is negligible compared with the plastic energy absorbed by the structure. Prager and Hopkins [1] were among the first to investigate the load carrying capacities of a circular plate made of perfectly plastic material which obeys the yield condition of Tresca and the associated flow rule. They assume small displacements and neglect membrane forces. The impulse is in the form of a uniformly distributed load which is applied suddenly and removed after a certain time interval. However, later studies [2] have indicated that membrane forces do dominate over the bending moments during the final stages of deformation. In their experimental work Griffith and Vanzent [3] have shown that the load carrying capacities of circular membranes are increased for large intensity loads of short duration.

Wang and Hopkins [4] have also neglected membrane forces in their analysis of circular plates. Pezyna's [5] investigation reveals that the pulse shape has very little effect on the final deformation pattern of a plate. Boyd [6] has used a numerical technique to solve the resulting

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differential equation of dynamic deformation of a circular membrane. Boyd's analysis was based on the deformation theory of plasticity. He therefore, neglects the effects of any possible unloading of the material during the deformation process. The permanent central deflections obtained by Boyd [6] and Witmer et. al [7] are found to be in close agreement. This seems to indicate that the rigid plastic assumption is adequate when membrane stresses are included in the analytical model.

In all the earlier investigations reviewed above only the effects of either bending moments or membrane forces were considered. Recently Jones [8] has examined the combined effects of membrane forces and bending moments on the behavior of circular plates loaded dynamically and for the case of large displacements. However, a simplified yield condition involving no-interaction between force and moment was employed by Jones. Florence [2] has investigated experimentally the behavior of simply supported circular plates subjected to uniformly distributed impulses. His results give ample evidence of the fact that membrane forces play a very important role in large deformation of circular plates.

As the rigid-plastic idealization is not appropriate for strain hardening materials, a more realistic method of analysis is required. In this spirit Witmer et al [9-11] have extensively studied the axisymmetric responses of beams, rings, circular plates and shells subjected to time dependent loads. Their finite difference method of solving the dynamic equations account for elastic, perfectly plastic, strain hardening and strain rate behavior of the material. However, because of the small time interval that must be used, considerable computing time is required even on a fast digital computer in solving a problem. Cox and Morland [12] have discussed the effect of a uniformly distributed rectangular pressure pulse

on a simply supported square plate. Their analysis neglected elastic strain, work hardening and strain rate effects. To simplify the mathematical analysis, a modified Tresca's yield criterion was used by them. Recently Jones et al [13] experimentally investigated the effect of uniformly distributed velocities on fully clamped rectangular plates.

It is well known that variational principles have been widely used for solving static problems of infinitesimal and finite plasticity [14-17]. A parallel minimum principle in dynamic plasticity has been developed by Tamuzh [18] for rigid-plastic bodies involving infinitesimal deformation. It was noted by Lee and Ni [19] that the principle is based on a concept of Gibbs and Appell [20], in classical mechanics, in employing finite variations in accelerations in formulating a minimum principle. Lee and Ni [19] have advanced the concept to establish an absolute minimum principle in dynamics of elastic-plastic continua subject to finite deformations. The minimum principle has the advantage of circumventing some of the difficulties in treating the loading and unloading material response. This minimum principle is employed to solve the titled problem.

The present study is concerned with the analysis of the dynamic response of a rectangular plate subjected to impulsive loading. The impulse is applied instantaneously at zero time and then decreases linearly with time. The response depends on the non-conservative, strain rate sensitivity and strain hardening properties of the material. A numerical procedure based on the minimum principle is developed to determine the maximum and the permanent displacements of a rectangular plate. The minimum principle, which is derived from the basic equation of motion, is written in terms of Piola-Kirchhoff's stresses, accelerations and Lagrangian strains. Kantorovich's method [21] is used to determine the time dependent

deformation profile of the plate. In conserving computing time, a sandwich plate idealization is employed in the numerical procedure.

KINEMATICS

Because the deformed configuration due to large deflections differs considerably from the initial configuration, the necessity of specifying whether the stresses and strains are measured with respect to the original configuration or with respect to the deformed configuration arises. If the original state of the material is homogeneous and stress free, it is convenient to express the dynamic equations in terms of the original state. The strain tensor, E_{IJ} referred to the initial configuration, is called Lagrangian strain tensor and is often referred to as the strain tensor in Lagrangian coordinates. Analogously, the strain tensor, e_{ij} referred to the deformed configuration is known as Almansi's strain tensor. In the following, we shall choose the Lagrangian descriptions of stress and strain.

Consider both the original and the deformed configurations of a body. Let two systems of coordinates X_I $(I = 1, 2, 3)$ and $x_i (i = 1, 2, 3)$ be chosen to describe the initial and the deformed configurations respectively. If we use the same rectangular cartesian coordinate system to describe both the original and the deformed configurations, then the Lagrangian strain tensor E_{IJ} may be defined in terms of the displacements, $U_I = x_i - X_I$, such that

$$E_{IJ} = \frac{1}{2} (U_{I,J} + U_{J,I} + U_{L,I} U_{L,J}) \tag{1}$$

Here, a partial differentiation of a variable with respect to X_K is designated as $( ),K$.

Now consider a rectangular plate of length 2a, breadth 2b, and thickness h. Let the components of the displacement vector $\vec{U}$ of any particle

originally located at (X, Y, Z) along the three mutually perpendicular directions, be $\bar{u}$, $\bar{v}$, and w in the direction normal to the plate respectively. It is assumed that the plate is, thin, the magnitudes of the deflections are of the same order as the plate thickness, h. Also, it is assumed that the displacements vary linearly through the thickness, and is independent of the distance z from the middle surface. Hence, the displacement components at any point may be expressed in terms of the displacements and their derivatives of the middle surface as*

$$
\begin{aligned}
\bar{u} &= u - z w_{,x} \\
\bar{v} &= v - z w_{,y}
\end{aligned}
\tag{2}
$$

in which u and v are the displacements along the directions x and y respectively of any point on the middle surface of the plate. Then, the second time derivatives of the strain components are

$$
\begin{aligned}
\ddot{E}_{XX} &= \ddot{\bar{u}}_{,x} + (\ddot{\bar{u}}_{,x}\bar{u}_{,x} + \dot{\bar{u}}_{,x}^{2} + \dot{\bar{v}}_{,x}^{2} + \bar{v}_{,x}\ddot{\bar{v}}_{,x} + w_{,x}\ddot{w}_{,x} + \dot{w}_{,x}^{2}) \\
\ddot{E}_{YY} &= \ddot{\bar{v}}_{,y} + (\dot{\bar{u}}_{,y}^{2} + \bar{u}_{,y}\ddot{\bar{u}}_{,y} + \dot{\bar{v}}_{,y}^{2} + \bar{v}_{,y}\ddot{\bar{v}}_{,y} + w_{,y}\ddot{w}_{,y} + \dot{w}_{,y}^{2}) \\
\ddot{E}_{XY} &= \frac{1}{2}(\ddot{\bar{u}}_{,y} + \ddot{\bar{v}}_{,x} + \ddot{\bar{u}}_{,x}\bar{u}_{,y} + 2\dot{\bar{u}}_{,x}\dot{\bar{u}}_{,y} + \bar{u}_{,x}\ddot{\bar{u}}_{,y} \\
&\quad + 2\dot{\bar{v}}_{,x}\dot{\bar{v}}_{,y} + \bar{v}_{,x}\ddot{\bar{v}}_{,y} + \ddot{\bar{v}}_{,x}\bar{v}_{,y} + 2\dot{w}_{,x}\dot{w}_{,y} + \ddot{w}_{,x}w_{,y} + w_{,x}\ddot{w}_{,y})
\end{aligned}
\tag{3}
$$

Here a dot above any term indicates partial differentiation with respect to time. The strain-accelerations given by Eq. (3) will be employed in the following minimum principle.

MINIMUM PRINCIPLE

Consider a body of a continuum occupying in its natural state a region

---

*Here, the subscripts x, y, z are referred to X, Y, Z coordinates.

V_o and bounded by a piecewise smooth surface A. The body is subjected to time dependent body force F_M (per unit mass) and Lagrangian surface traction (per unit initial area) T_M over that part of the initial surface area A_T. At time t, let $\{U_K\}$ be the displacement vector of a particle of the body which has an initial position of $\{X_K\}$ in a rectangular Cartesian coordinate system. The displacements are prescribed over a part of the boundary surface, A_U. The deformation of the body may be described in terms of the Lagrangian strain tensor, E_{KL}, defined by equation (1). Furthermore the Lagrangian strains E_{KL} may be expressed as the sum of two parts: elastic strain, E'_{KL}, and plastic strain E''_{KL}. It is postulated that the constitutive relationships, in terms of Piola-Kirchhoff stress tensor, S_{KL}, may be strain velocity dependent but are not influenced by strain accelerations. In other words, it is assumed that

$$S_{KL} = S_{KL}\,(E'_{MN},\ E''_{MN},\ \dot{E}''_{MN},\ \theta) \tag{4}$$

where θ is the temperature and $\dot{E}''_{MN}$ is the velocity rate of plastic straining. The Piola-Kirchhoff stresses satisfy the boundary conditions

$$S_{KL}\,(\delta_{ML} + U_{M,L})\,N_K = T_M \quad \text{on} \quad A_T \tag{5}$$

where N_K is the outward unit normal to A and δ_{ML} is the Kronecker symbol.

It has been shown [19] that the true acceleration field, $\ddot{U}_M = \dfrac{D^2 U_M}{Dt^2}$, of the body, which has known or predetermined displacement and velocity fields at time t, is distinguished from all kinematically admissible ones by having the minimum value of the following functional

$$J = \int_{V_o} S_{KL}\ \ddot{E}_{KL}\ d\,V_o + \frac{1}{2}\int_{V} \rho_o\ \ddot{U}_M^{\,2}\ dV_o - \int_{A_T} T_M\ \ddot{U}_M\ dA - \int_{V_o} \rho_o\ F_M\ \ddot{U}_M\ d\,V_o \tag{5}$$

where ρ_o is the initial mass density. The minimum principle is valid for continuous as well as sectionally discontinuous acceleration fields. Ordinarily, it is sufficient to use the first variation with respect to the acceleration, $\delta_{acc}\ J = 0$, to establish governing equations or to solve a problem by a direct method of variational calculus.

CONSTITUTIVE EQUATIONS

The stress-strain relationships derived here are for those materials which are isotropic, inelastic and work hardening. Furthermore it is assumed that the material behaves isothermally. It is known that for the mechanical behavior of the material in the inelastic range, the state of stress or strain may be represented by a point in a nine dimensional stress or strain space respectively. A system of loading is considered as a path in this stress space and the corresponding deformation history as a path in the strain space.

A basic assumption is made that there exists a scalar function, called a yield function or loading function F, which depends on the states of stress and strain and the history of loading. The equation $F = 0$ represents a closed surface in the nine dimensional stress space. Inelastic deformation can occur only when $F = 0$. The condition $F < 0$ indicates that no permanent deformation is possible and the condition $F > 0$ has no given physical meanings. It is relatively easy to determine the yield stress for a material subjected to an axial tensile load. Therefore, it is desirable to express mathematically

a general stress-strain relationship in terms of uniaxial mechanical properties.

In a simple tensile test, work hardening means that the stress is a monotonically increasing function of increasing strain. For a general state of stress, Drucker's definition of work hardening implies that energy cannot be extracted from the material in the process of applying and removing a system of forces acting on a body. The following conditions are a consequence of Drucker's hypothesis: a) The yield surface is convex , and b) the plastic strain increment vector is normal to the loading surface.

Using these conditions, Lee and Murphy [14] have obtained a relationship between Lagrangian strain increments and Piola-Kirchhoff's stress increments as

$$\dot{E}''_{IJ} = G \frac{\partial F}{\partial S_{IJ}} \frac{\partial F}{\partial S_{KL}} \dot{S}_{KL}, \text{ for } F=0 \text{ and } dF>0 \tag{7}$$

and

$$\dot{E}''_{IJ} = 0 \qquad \text{for } F<0 \text{ or } dF \leq 0 \tag{8}$$

where $F = J_2 - K^2$.

Here G is a scalor proportionality function which depends on the current stress, plastic strain and the loading history. For a strain-rate insentive material, K^2 is a constant which is equal to the maximum value of J_2 that has occurred until the current deformation state. J_2 is the second invariant of the stress deviation tensor S'_{ij} with

$$J_2 = \frac{1}{2} S'_{KL} \; S'_{KL}$$
$$S'_{KL} = S_{KL} - \frac{1}{3} S_{MM} \; \delta_{KL} \tag{9}$$

The expressions $dF>0$, $dF = 0$ and $dF<0$ imply respectively, loading,

neutral loading and unloading of the material in relationship to the yield surface, characterized by the loading function. For a given state of stress on the yield surface, either loading, unloading or neutral loading takes place according to whether stress increment vector is directed outward, inward or along a tangent to the loading surface. Available experimental results indicate that the Mises' yield function or loading function leads to a good prediction of the initial yielding of an isotropic material. Furthermore, in the absence of Banschinger's effect and in a case of relatively small strains, the Mises' function give also a good prediction of the subsequent yielding. Usually, the Mises' function has been expressed in terms of the true stress. For a case of small strain, the difference between the true and Piola-Kirchoff stress tensors is relatively small. For the thin plate problem, it is expected the foregoing conditions prevail. Employing the concept of isotropic hardening and assuming that the scalar function G is a function of J_2 only, G may be determined from the results of a simple tension test. It may be shown that [14]

$$G = \frac{3}{4J_2}\left(\frac{1}{E_t} - \frac{1}{E}\right) \quad \text{for } \dot{J}_2 > 0$$

and (10)

$$G = 0 \qquad \text{for } \dot{J}_2 \leq 0$$

where E_t and E are the tangent and elastic moduli of the material respectively. Furthermore the general state of stress is related to the simple tensile stress, S, by the relation

$$J_2 = S^2/3 \tag{11}$$

In simple tension, yielding occurs when S reaches the yield stress S_0.

In the case of a thin rectangular plate, the rate of the second invariant

of the stress deviation tensor may be written in terms of the Piola- Kirchhoff stresses as

$$\dot{J}_2 = \frac{1}{3}\left[\dot{S}_{XX}\left(2S_{XX}-S_{YY}\right)+\dot{S}_{YY}\left(2S_{YY}-S_{XX}\right) + 6\,\dot{S}_{XY}S_{XY}\right] \tag{12}$$

The elastic strain rate $\dot{E}'_{KL}$ is given by the Hooke's law,

$$\dot{E}'_{KL} = \frac{1+\nu}{E}\dot{S}_{KL} - \frac{\nu}{E}\dot{S}_{MM}\,\delta_{KL} \tag{13}$$

where ν is the Poisson's ratio. Employing Eqs. (7) to (13) and the assumption of plane stress, the stress-strain relationship for the thin plate may be written as

$$\begin{aligned}\dot{E}_{XX} &= C_{11}\dot{S}_{XX} + C_{12}\dot{S}_{XY} + C_{13}\dot{S}_{YY}\\ \dot{E}_{XY} &= C_{21}\dot{S}_{XX} + C_{22}\dot{S}_{XY} + C_{23}\dot{S}_{YY}\\ \dot{E}_{YY} &= C_{31}\dot{S}_{XX} + C_{32}\dot{S}_{XY} + C_{33}\dot{S}_{YY}\end{aligned} \tag{14}$$

where for $\dot{J}_2 > 0$ and $J_2 = K^2$

$$\begin{aligned}C_{11} &= \frac{1}{E} + \frac{G}{9}\left(2S_{XX}-S_{YY}\right)^2\\ C_{12} &= 2S_{XY}\,G\,\frac{\left(2S_{XX}-S_{YY}\right)}{3}\\ C_{13} &= -\frac{\nu}{E} + G\,\frac{\left(2S_{XX}-S_{YY}\right)\left(2S_{YY}-S_{XX}\right)}{9}\\ C_{21} &= C_{12}\\ C_{22} &= \frac{1+\nu}{E} + 2S^2_{XY}\,G\\ C_{23} &= 2S_{XY}\,G\,\frac{\left(2S_{YY}-S_{XX}\right)}{3}\end{aligned} \tag{15}$$

$$C_{31} = C_{23}$$

$$C_{32} = C_{23} \qquad (15)$$

$$C_{33} = \frac{1}{E} + G \frac{\left(2S_{YY} - S_{XX}\right)^2}{9}$$

and

for $\dot{J}_2 \leq 0$ and $J_2 = K^2$ or $\dot{J}_2 > 0$ and $J_2 < K^2$

$$C_{11} = \frac{1}{E},\ C_{12} = 0,\quad C_{13} = -\frac{\nu}{E},$$

$$C_{21} = C_{12},\ C_{22} = \frac{1+\nu}{E},\ C_{23} = 0, \qquad (16)$$

$$C_{31} = C_{13},\ C_{32} = C_{23},\ C_{33} = \frac{1}{E}$$

In the use of the minimum principle, it is convenient to express stress rates in terms of strain rates. Thus, the inverted form of equation (14) may be given as

$$\dot{S}_{XX} = F_{11}\,\dot{E}_{XX} + F_{12}\,\dot{E}_{XY} + F_{13}\,\dot{E}_{YY}$$

$$\dot{S}_{XY} = F_{21}\,\dot{E}_{XX} + F_{22}\,\dot{E}_{XY} + F_{23}\,\dot{E}_{YY} \qquad (17)$$

$$\dot{S}_{YY} = F_{31}\,\dot{E}_{XX} + F_{32}\,\dot{E}_{XY} + F_{33}\,\dot{E}_{YY}$$

where

$$F_{11} = \left(C_{22}\,C_{33} - C_{23}\,C_{32}\right)/D$$

$$F_{12} = -\left(C_{12}C_{33} - C_{13}C_{32}\right)/D$$

$$F_{13} = \left(C_{12}\,C_{23} - C_{13}\,C_{22}\right)/D \qquad (18)$$

$$F_{21} = F_{12}$$

$$F_{22} = \left(C_{11}\,C_{33} - C_{13}\,C_{31}\right)/D$$

$$F_{23} = -\left(C_{11}C_{23} - C_{13}C_{21}\right)/D$$

$$F_{31} = F_{13}$$

$$F_{32} = F_{33} \tag{18}$$

$$F_{33} = \left(C_{11}C_{22} - C_{12}C_{21}\right)/D$$

and

$$D = C_{11}\left(C_{22}C_{33} - C_{23}C_{32}\right) - C_{12}\left(C_{21}C_{33} - C_{23}C_{31}\right) + C_{13}\left(C_{21}C_{32} - C_{31}C_{22}\right) \tag{19}$$

In dynamic plasticity, the mechanical stress-strain properties of some materials are significantly affected by the rate of straining. The immediate effect of straining rate is that the yield stress is effectively increased. In order to take into account the effects of the strain rate on the yield stress, the following strain rate formula which relates the yield stress at any instant of time to the static yield stress will be employed [11].

$$S_o^* = S_o\left(1 + \left|\frac{\Delta\epsilon/\Delta t}{\bar{D}}\right|^{1/\underline{P}}\right) \tag{20}$$

where S_o^* is the yield stress at any instant of time and S_o is the static yield stress. $\underline{P}$ and $\bar{D}$ are material constants determined by experiments [10]. ϵ is the "effective" strain rate, and can be expressed in terms of the generalized strain rates as

$$\dot{\epsilon} = \left(\frac{4}{3}\right)^{1/2}\left(\dot{E}_{XX}^2 + \dot{E}_{XY}^2 + \dot{E}_{XX}\dot{E}_{YY} + \dot{E}_{XY}^2\right)^{1/2} \tag{21}$$

DERIVATION OF FUNCTIONAL

For the case of a thin flat plate subjected to an uniformly distributed transverse impulsive load, by neglecting body forces, equation (6) reduces to the following form

$$J = \int_{V_o} \frac{\rho_o}{2}\left(\ddot{u}^2 + \ddot{v}^2 + \ddot{w}^2\right) dV_o - \int_{A_T} p\,(t)\ \ddot{w}\ d\,A_T$$
$$+ \int_{V_o} \left(S_{XX}\,\ddot{E}_{XX} + 2\,S_{XY}\,\ddot{E}_{XY} + S_{YY}\,\ddot{E}_{YY}\right) d\,V_o \qquad (22)$$

where p(t) represents the uniformly distributed transverse load applied to the plate. It is seen that the minimum principle is expressed entirely in terms of the current state of stress, strain-acceleration and acceleration of the body. The current state of stress is not only a function of position but also depends on the loading history. This makes the analytical integration of equation (22) very difficult to overcome. This difficulty, an approximate solution, the Kantororich's method may be employed or the functions w (x, y, t), u(x, y, t) and v (x, y, t) may be represented by

$$\bar{w}\,(x, y, t) = \sum_{m, n=1}^{M, N} d_{mn}(t)\ X_m\,(x)\ Y_n(y)$$
$$u\,(x, y, t) = \sum_{m, n=1}^{M, N} e_{mn}\,(t)\,\varphi_m\,(x)\ Y_n(y) \qquad (23)$$
$$v\,(x, y, t) = \sum_{m, n=1}^{M, N} f_{mn}\,(t)\ X_m(x)\ \Psi_n(y)$$

with

$m = 1, 2, 3 \ldots\ M$

$n = 1, 2, 3, \ldots\ N$

Here $X_m(x)\ Y_n(y)$, $\varphi_m(x)$, and $\Psi_n(y)$ are chosen coordinate functions, each of which satisfies prescribed geometric boundary condtions and any other

special characteristic of the solution such as a condition of symmetry. The coefficients $d_{mn}(t)$, $e_{mn}(t)$ and $f_{mn}(t)$ are unknown functions of time, t to be determined by the minimization of the functional J with respect to $\ddot{d}_{mn}(t)$, $\ddot{e}_{mn}(t)$, and $\ddot{f}_{mn}(t)$. In general, to obtain an exact solution, an infinite number of coordinate functions are needed. However, in practice only a finite number of these coordinate functions can be considered. This number depends not only on the magnitude of the computation involved, but on the desired accuracy of the solution.

IMPULSIVE LOADING OF AN INELASTIC FLAT PLATE WITH EDGE CONSTRAINTS

For verification the foregoing approach is applied to the analysis of the dynamic response of an inelastic rectangular plate subjected to impulsive loading. If I is the applied impulse per unit area of the plate, it can be approximated by a simple triangular time history starting at pressure p_o and reaching zero at a time $\bar{t}$ such that the area under pressure-time curve is equal to this applied impulse. In other words

$$\begin{aligned} p\,(x,y,t) &= 0 && t<0 \\ p\,(x,y,t) &= p_o \frac{(\bar{t}-t)}{\bar{t}} && t\le\bar{t} \\ p\,(x,y,t) &= 0 && t>\bar{t} \end{aligned} \tag{24}$$

where $p(x,y,t)$ is the pressure at any time t. The tangential acceleration components $\ddot{\bar{u}}$ and $\ddot{\bar{v}}$ of a point away from the middle surface are expressed in terms of the middle surface components $\ddot{u}$ and $\ddot{v}$ by differentiating equation (2) twice with respect to time they are

$$\begin{aligned} \ddot{\bar{u}} &= \ddot{u} - z\,\ddot{w}_{,x} \\ \ddot{\bar{v}} &= \ddot{v} - z\,\ddot{w}_{,y} \end{aligned} \tag{25}$$

Furthermore the spatial coordinate functions in equations (23) are assumed to be continuous and differentiable. By differentiating equation (2) twice with respect to time and once with respect to space coordinates one obtains

$$
\begin{aligned}
\ddot{\bar{u}}_{,x} &= \ddot{u}_{,x} - z\,\ddot{w}_{,xx} \\
\ddot{\bar{u}}_{,y} &= \ddot{u}_{,y} - x\ddot{w}_{,xy} \\
\ddot{\bar{v}}_{,x} &= \ddot{v}_{,x} - z\,\ddot{w}_{,xy} \\
\ddot{\bar{v}}_{,y} &= \ddot{v}_{,y} - z\,\ddot{w}_{,yy}
\end{aligned}
\tag{26}
$$

Using equations (3), (25) and (26), we can express J given by equation (22) in terms of displacements and their time derivatives as

$$
\begin{aligned}
J = &\int_{V_o} \frac{\rho_o}{2} \left\{ (\ddot{u} - z\,\ddot{w}_{,x})^2 + (\ddot{v} - z\ddot{w}^2 \right\} dV_o \\
&- \int_{A_T} p\,(t)\,\ddot{w}\; dA_T \\
&+ \int_{V_o} \Big\{ S_{xx}(\ddot{\bar{u}}_{,x} + \ddot{\bar{u}}_{,x}\bar{u}_{,x} + \dot{\bar{u}}_{,x}{}^2 \\
&+ \dot{\bar{v}}_{,x}{}^2 + \bar{v}_{,x}\,\ddot{\bar{v}}_{,x} + w_{,x}\,\ddot{w}_{,x} + \dot{w}_{,x}{}^2) \\
&+ 2S_{xy}(\ddot{\bar{u}}_{,y} + \ddot{\bar{v}}_{,x} + \ddot{\bar{u}}_{,x}\,\bar{u}_{,y} + 2\dot{\bar{u}}_{,x}\,\dot{\bar{u}}_{,y} \\
&+ \bar{u}_{,x}\,\ddot{\bar{u}}_{,y} + 2\dot{\bar{v}}_{,x}\,\dot{\bar{v}}_{,y} + \bar{v}_{,x}\,\ddot{\bar{v}}_{,y} + \ddot{\bar{v}}_{,x}\,\bar{v}_{,y} \\
&+ 2\dot{w}_{,x}\,\dot{w}_{,y} + \ddot{w}_{,x}\,w_{,y} + w_{,x}\,\ddot{w}_{,y}) + S_{yy}\,(\ddot{\bar{v}}_{,y} \\
&+ \dot{\bar{u}}_{,y}{}^2 + \bar{u}_{,y}\,\ddot{\bar{u}}_{,y} + \dot{\bar{v}}_{,y}{}^2 + \bar{v}_{,y}\,\ddot{\bar{v}}_{,y} + w_{,y}\,\ddot{w}_{,y} \\
&+ \dot{w}_{,y}{}^2) \Big\}\; dV_o
\end{aligned}
\tag{27}
$$

The volume integral J may be expressed as a function of acceleration coefficients by substituting equations (23) and (25) into the equation (27). The variation of J with respect to $\ddot{u}_i$ implies the variation of J with respect to the acceleration coefficients $\ddot{d}$, $\ddot{e}$ and $\ddot{f}$. The variation of J with respect to each coefficient results in an expression which when equated to zero, results in the following three sets of quasi-linear algebraic equations.

$$\int_{V_o} \Big[\sum_m \sum_n \rho_o \Big\{ (\ddot{e}_{mn} \varphi_m Y_n - z\, \ddot{d}_{mn} X_{m,x} Y_n)(-z X_{i,x} Y_j)$$

$$+ (\ddot{f}_{mn} X_m \psi_n - z\, \ddot{d}_{mn} X_m Y_{n,y})(-z X_i Y_{j,y})$$

$$+ \ddot{d}_{mn} X_m Y_n X_i Y_j \Big\} \Big] dV_o \qquad \begin{array}{l} m = 1, 2, 3, \ldots \\ n = 1, 2, 3, \ldots \\ i = 1, 2, 3, \ldots m \\ j = 2, 3, \ldots\ldots n \end{array}$$

$$- \int_{A_o} p(t)\, X_i Y_j \, dA_o$$

$$+ \int_{V_o} I_{ij1}\, dV_o = 0 \qquad (28)$$

$$\int_{V_o} \Big[\sum_m \sum_n \rho_o \Big\{ (\ddot{e}_{mn} \varphi_m Y_n - z\, \ddot{d}_{mn} X_{m,x} Y_n)\, \varphi_i Y_j \Big\} \Big] dV_o$$

$$+ \int_{V_o} I_{ij2}\, dV_o = 0 \qquad (29)$$

$$\int_{V_o} \Big[\sum_m \sum_n \rho_o \Big\{ (\ddot{f}_{mn} X_m \psi_n - z\, \ddot{d}_{mn} X_m Y_{n,y})\, X_i \psi_j \Big\} \Big] dV_o$$

$$+ \int_{V_o} I_{ij3}\, dV_o = 0 \qquad (30)$$

where

$$I_{ij1} = \Big[S_{xx} \Big\{ -z\, X_{i,xx} Y_j \,(1 + \overline{u}_{,x}) + X_{i,x} \,(-z\, Y_{j,y}\, \overline{v}_{,x} + w_{,x}\, Y_j) \Big\} + S_{xy} \Big\{ -z\, X_{i,x} Y_{j,y} (2 + \overline{u}_{,x} + \overline{v}_{,y}) - z\, X_{i,xx}\, Y_j\, \overline{u}_{,y} - z\, X_i\, Y_{j,yy}\, \overline{v}_{,x} + w_{,y} X_{i,x}\, Y_j + w_{,x}\, X_i\, Y_{j,y} \Big\} + S_{yy} \Big\{ -z\, X_i\, Y_{j,yy} (1 + \overline{v}_{,y}) - z\, Y_{j,y}\, \overline{u}_{,y}\, X_{i,x} + Y_{j,y}\, w_{,y}\, X_i \Big\} \Big] \tag{31}$$

$$I_{ij2} = \Big[S_{xx} \Big\{ \varphi_{i,x}\, Y_j\, (1 + \overline{u}_{,x}) \Big\} + S_{xy} \Big\{ \varphi_i\, Y_{j,y}\, (1 + \overline{u}_{,x}) + \varphi_{i,x}\, Y_j\, \overline{u}_{,y} \Big\} + S_{yy}\, (\overline{u}_{,y}\, \varphi_i\, Y_{j,y}) \Big] \tag{32}$$

$$I_{ij3} = \Big[S_{xx}\, (\overline{v}_{,x}\, X_{i,x}\, \psi_j) + S_{xy} \Big\{ \psi_j\, X_{i,x}\, (1 + \overline{v}_{,y}) + \overline{v}_x\, X_i\, \psi_{j,y} \Big\} + S_{yy} \Big\{ X_i\, \psi_{j,y}\, (1 + \overline{v}_{,y}) \Big\} \Big] \tag{33}$$

The solutions of the quasi-linear algebraic equations are obtained by employing a finite difference scheme together with numerical step-by-step method. The plate is idealized into a number of load carrying layers separated by a core material. The stress variation through each layer is assumed to be negligible. The only function of the core is to maintain

an equal separation of the layers. There is an even number of layers and they are of equal thickness. The spacing between the discrete layers may be obtained by assuming both the idealized model and the actual plate exhibit equal elastic bending stiffnesses. Thus

$$\bar{b} = \frac{3h^2}{20} \qquad k = 4$$
$$\bar{b} = \frac{h^2}{12} \qquad k = 2 \qquad (34)$$

where k denotes the number of layers and $2\bar{b}$ is the distance between the two extreme layers. The volume integrals in Eqs. (28)-(30) are reduced to surface integrals by integrating them first with respect to the z coordinate. The integration along the z coordinate may be performed by multiplying the integrand by a factor h/k and summing the integrand over the number of layers considered.

The surface integrations are carried out as follows. Let (M-1) and (N-1) be the number of segments along the x and y axes respectively. M and N then represent the number of stations along the two coordinate axes. The coordinates (x, y) of any station (p, q) are given by

$$x = \frac{p-1}{M-1}\ a \qquad p = 1, 2, \ldots M$$
$$y = \frac{q-1}{N-1}\ b \qquad q = 1, 2, \ldots N \qquad (35)$$

where a and b represent the semi-lateral or lateral dimensions of the plate, depending upon whether the origin of the coordinate system is at the center or at the corner of the rectangular plate. If f(x, y) denotes the integrand in a surface integral, the integration is performed by using the following expression

$$\int_{A_T} f(p,q)\, dA_T = \frac{ab}{4(M-1)(N-1)} \Big[f(1,1) + F(1,N) + f(M,1) + f(M,N) + 2\sum_{q=2}^{q=N-1} \{ f(1,q) + f(M,q) \}$$

$$+ 2\sum_{p=2}^{p=M-1} \{ f(p,1) + f(p,N) \} + 4\sum_{p=2}^{p=M-1}\sum_{q=2}^{q=N-1} f(p,q) \Big] \qquad (36)$$

By the solutions of the quasi-linear algebraic equations, the accelerations and hence the displacement rates are determined. The strain and stress increments can be calculated. The new displacements, stresses and strains are then computed. A time-displacement history is maintained throughout the motion of the plate until it comes to rest and then the permanent displacements of the plate are recorded.

In this analysis the detailed computations are carried out for the following two cases:

a) All edges of the plate are simply supported.
b) All edges of the plate are fixed.

The boundary conditions for the first case are satisfied by the following chosen coordinate functions (the center of the plate is taken as the origin of the coordinate system).

$$X_m = \cos\frac{\pi(2m-1)x}{2a} \qquad m = 1, 2, 3 \ldots$$

$$Y_n = \cos\frac{\pi(2n-1)y}{2b} \qquad n = 1, 2, 3 \ldots$$

$$\varphi_m = \sin\frac{m\pi x}{a} \qquad m = 1, 2, 3 \ldots$$

$$\psi_n = \sin\frac{n\pi y}{b} \qquad n = 1, 2, 3 \ldots \qquad (37)$$

It is to be noted that the functions X_m, Y_n, $X_{m,x}$, $Y_{n,y}$, φ_m and Ψ_n have certain properties. Substitution of Eqs. (37) into Eqs. (28), (29) and (30) yields the coefficients

$$d_{ij} = \frac{P(t)\,\dfrac{4ab}{\pi^2(2n-1)(2m-1)ab}\,(-1)^{m+n} - \int_{V_o} I_{ij_1}\, d\,V_o}{\rho_o\left[\displaystyle\sum_{lk=1}^{lk=k} \left\{ z^2 \frac{\pi^2}{16a}\,(2m-1)^2\, b + z^2\,\frac{\pi^2}{16b}\,(2n-1)^2\, a \right\} + \frac{ab}{4}\, h\right]}$$

$$\ddot{e}_{ij} = \frac{-\int_{V_o} I_{ij_2}\, dV_o}{h\rho_o\, \dfrac{ab}{4}}$$

and

$$\ddot{f}_{ij} = \frac{-\int_{V_o} I_{ij_3}\, dV_o}{h\rho_o\, \dfrac{ab}{4}} \qquad \begin{array}{l} i = 1, \ldots, m \\ j = 1, \ldots, n \end{array} \qquad (38)$$

where the symbol $\sum_{lk=1}^{lk=k}$ indicates the summation with respect to each of the load carrying layers located at z.

For the case of the clamped plate with one of the plate corners taken as the origin of the coordinate system, the edge conditions are satisfied by the coordinate functions

$$X_m = 1 - \cos\frac{2\pi m x}{2a} \qquad m = 1, 2, \ldots$$

$$Y_n = 1 - \cos\frac{2\pi n y}{2b} \qquad n = 1, 2, \ldots$$

$$\varphi_m = \sin\frac{m\pi x}{2a} \qquad m = 1, 2, \ldots$$

$$\Psi_n = \sin\frac{n\pi y}{2b} \qquad n = 1, 2, \ldots \qquad (39)$$

The function X_m and Y_n are not orthogonal. However, the functions $X_{m,x}$, $Y_{n,y}$, φ_m and Ψ_n are orthogonal. Now Eqs. (28), (29) and (30) by utilizing Eqs. (39) may be reduced to the following expressions

$$\sum_m \sum_n \rho_o \left[\sum_{Ik=1}^{Ik=k} \left\{ z^2 \ddot{d}_{in} \frac{\pi^2}{a} m^2 Y_{nj} + z^2 \ddot{d}_{mj} \frac{\pi^2}{b} n^2 X_{im} \right\} \right.$$

$$\left. + h \ddot{d}_{mn} X_{mi} Y_{nj} \right] - 4p(t)\ ab$$

$$+ \int_{V_o} I_{ij_1}\, dV_o = 0$$

$$\sum_n \rho_o h \ddot{e}_{in}\, a\, Y_{nj} + \int_{V_o} I_{ij_2}\, dV_o = 0$$

and

$$\sum_m \rho_o \ddot{f}_{mj}\, b\, X_{im} + \int_{V_o} I_{ij_3}\, dV_o = 0 \qquad \begin{matrix} i = 1, \dots, m \\ j = 1, \dots, n \end{matrix} \qquad (40)$$

where

$$X_{mi} = \int_o^{2a} X_m X_i\, dx$$

$$Y_{nj} = \int_o^{2b} Y_n Y_j\, dy \qquad (41)$$

The acceleration coefficients $\ddot{d}_{ij}$, $\ddot{e}_{ij}$, and $\ddot{f}_{ij}$ are obtained by solving the three sets of (mxn) simultaneous equations (40).

RESULTS AND DISCUSSION

Extensive numerical results have been obtained for a thin flat square plate (a = 12in., h= 1/8 and 3/8 in.) made of 61S-T6 aluminum alloy. The stress strain relationship may be approximated by the bilinear

representation with $E = 10 \times 10^6$ psi, $S_o = 40,000$ psi, and $E_t = 10 \times 10^4$ psi. The initial mass density is $\rho_o = 253 \times 10^{-6} lb/sec^2/in^4$.

In the analysis of inelastic simply supported beams, Balmer and Witmer [11] showed that the maximum differences between the responses for 2 layered and 4 layered beams was about 8%. In the case of a clamped beam, the differences were slightly larger. However, in this analysis practically identical inelastic responses were obtained for 2 and 4 layered plates. Moreover, without loss of occuracy, it is assumed that the in-plane displacements are infinitesimal.

For the selection of the mesh size, a number of trial runs indicated that 12 and 10 stations adequately represent the half span of a simply supported and a clamped square plate respectively.* The value of Δt has been chosen on the basis of numerical experiments and is taken to be equal to 25 microseconds. The input impulse considered here for 1/8" and 3/8" thick plates are 0.083 and 0.190 psi-sec. respectively. Experimentally these respective impulses were obtained due to a blast caused by detonation of 8.35 and 36 lbs charges of pentolite located centrally 7.7 and 9.7 feet from the face of the square plate. The observed peak pressures were 283 and 615 psi. These impulses are approximated by simply triangular time histories starting at the peak pressures and reaching zero at a time such that the associated impulses are equal to the values cited earlier.

In Figures 1 and 2, the respective response time history for simply supported and clamped square plates are presented. In Figure 1 the responses of a simply supported circular plate by two other analyses [6, 7] and the permanent experimental central deflection [7] are also shown. Because Boyd's analysis [6] did not include linear unloading,

* Trail runs also indicated that a nine term series in equation (23) was adequate to represent the acceleration of motion.

the deformation process was assumed to stop when maximum strains occurred in the material. In Figure 1, the horizontal line drawn from the maximum displacement indicates this effect. The maximum displacements reported by Boyd and the present analysis are in close agreement but differ from those obtained by Witmer [7]. The permanent displacements predicted by this analysis are in much closer agreement with the experimental results [7] than those of the other two analyses. Both Boyd and Witmer assumed perfectly plastic material in their investigations.

From Figures 1 and 2, it is observed that the plate reaches an equilibrium position soon after it passes through its maximum displacement. By that time the elastic energy has been completely recovered. The mean vibration amplitude following the maximum displacement are entirely inelastic. The plate now oscillates about its new equilibrium point until it finally comes to rest. The responses of the center of a simply supported and a built-in plate are almost identical.

Figures 3 and 4, present the deflection pattern for the two different edge conditions considered here. Initially it is seen that the deformation profile is in the form of a trapezoid with rounded corners. This confirms the initial trapizoidal profile which is generally assumed for displacements in the limit analysis of plates. Any further motion of the plate changes the deformation profile to a parabolic shape. As the plate approaches its permanent deformed shape, the deformation profile seems to follow the dome shape.

Deflection contours for different time intervals are shown in Figures 5 through 7. Contours marked (a) to (c) are for the simply supported plate and those marked (d) to (f) are for the built-in plate. These contours are far apart in the beginning of the motion but they become closer as the motion

proceeds. The circular shape of these contours indicates the similar behavior of a square plate of side 2a and a circular plate of diameter 2a.

In Figures 8 and 9, the membrane froces for the simply supported plate along the semi-span for t = 250 microseconds and for t = 750 microseconds are plotted. It is seen that the membrane forces are negligible at the beginning of the motion and become rather effective as the motion proceeds. That is, initially the bending modes dominate over the membrane forces and these roles are gradually reversed as the plate come to rest.

In table 1, the permanent deflections of a 3" x 5-1/6" plate for various applied impulses are compared with those determined experimentally by Jones [13]. The results of the present analysis are lower than those by Jones. The same difference is observed in the plots of the permanent deflections in Figures 10 and 12. This difference may be attributed to various factors. However, it is believed that the major factor may be that the experimental set-up might not precisely provide the fixed and clamped edge boundary conditions utilized in the present analysis. It is also to be noted that both the theoretical and experimental results indicate that the profile of the permanent deflection of a square plate subjected to uniform impulsive pressure has the shape of a truncated pyramid.

CONCLUDING REMARKS

A numerical procedure based on an absolute minimum principle has been developed for investigating the dynamic responses of inelastic rectangular plates at finite deformation. The minimum principle is based on the concept of finite variation in accelerations which circumvents certain difficulties in treating the stress-strain relationships encountered in using other principles involving virtual displacements or velocities.

The present approach is a general one since it takes into account the effect of strain hardening behavior and the strain rate sensitivity of the material. Which has also the advantage of employing the Kantorovich's method in that a comparatively large time increment may be used without meeting numerical instabilities in the integration process. The accuracy of the present numerical results has been verified by comparision with the existing analytical, numerical and experimental results obtained for plates under impulsive loadings.

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TABLE 1

PERMANENT CENTRAL DISPLACEMENTS OF BUILT-IN 6061-T6 PLATE

SL No.	h(in)	a(in)	b(in)	Impulse (psi-sec)	W(in) Present Theory	W(in) Jones [8]
1	.123	3	5-1/6	.1246	.26	.35
2	.123	3	5-1/6	.0871	.17	.22
3	.123	3	5-1/6	.1468	.34	.43

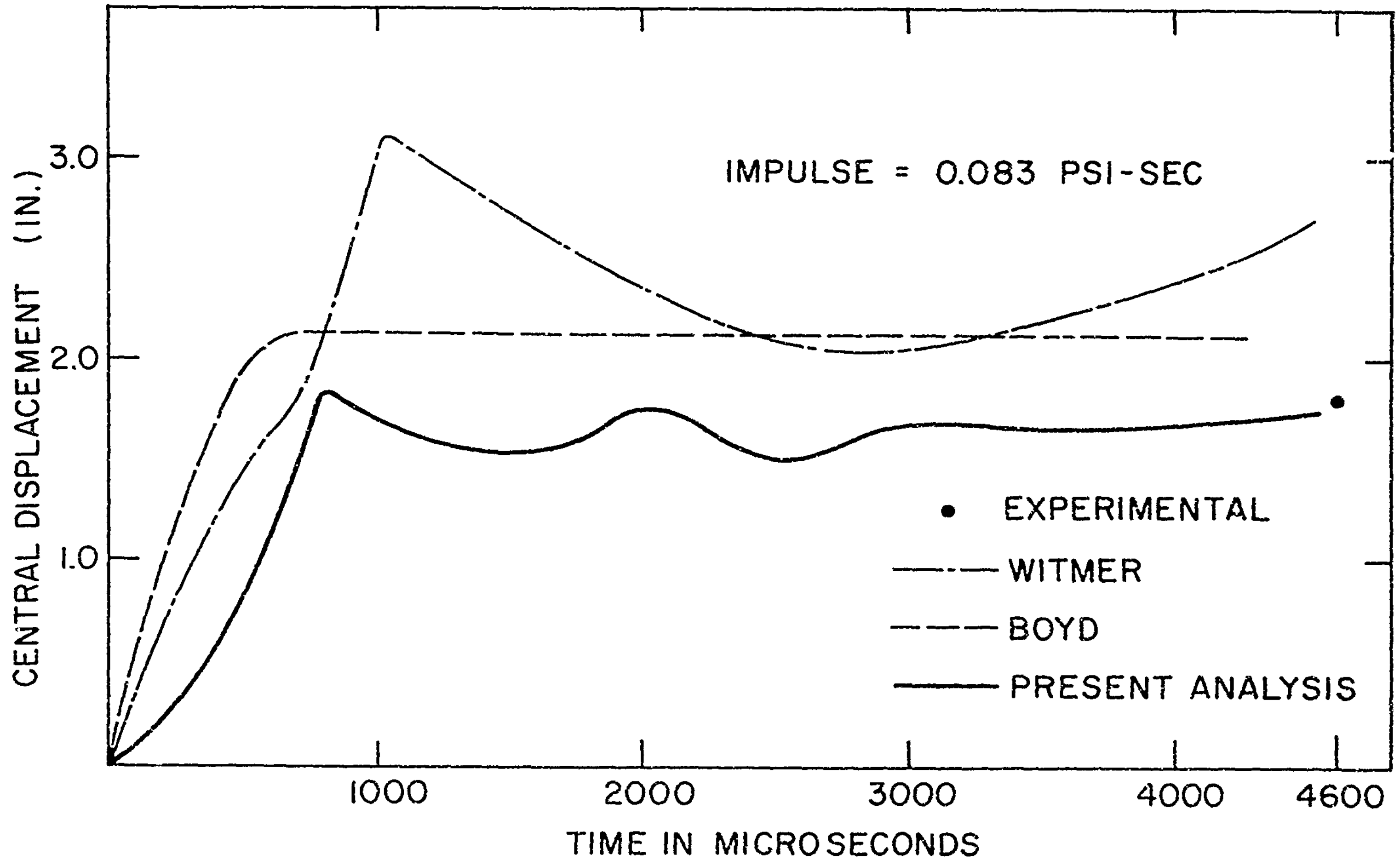

Fig. 1 Comparison of Theoretical Responses with Experimental Results for a Simply Supported Plate.

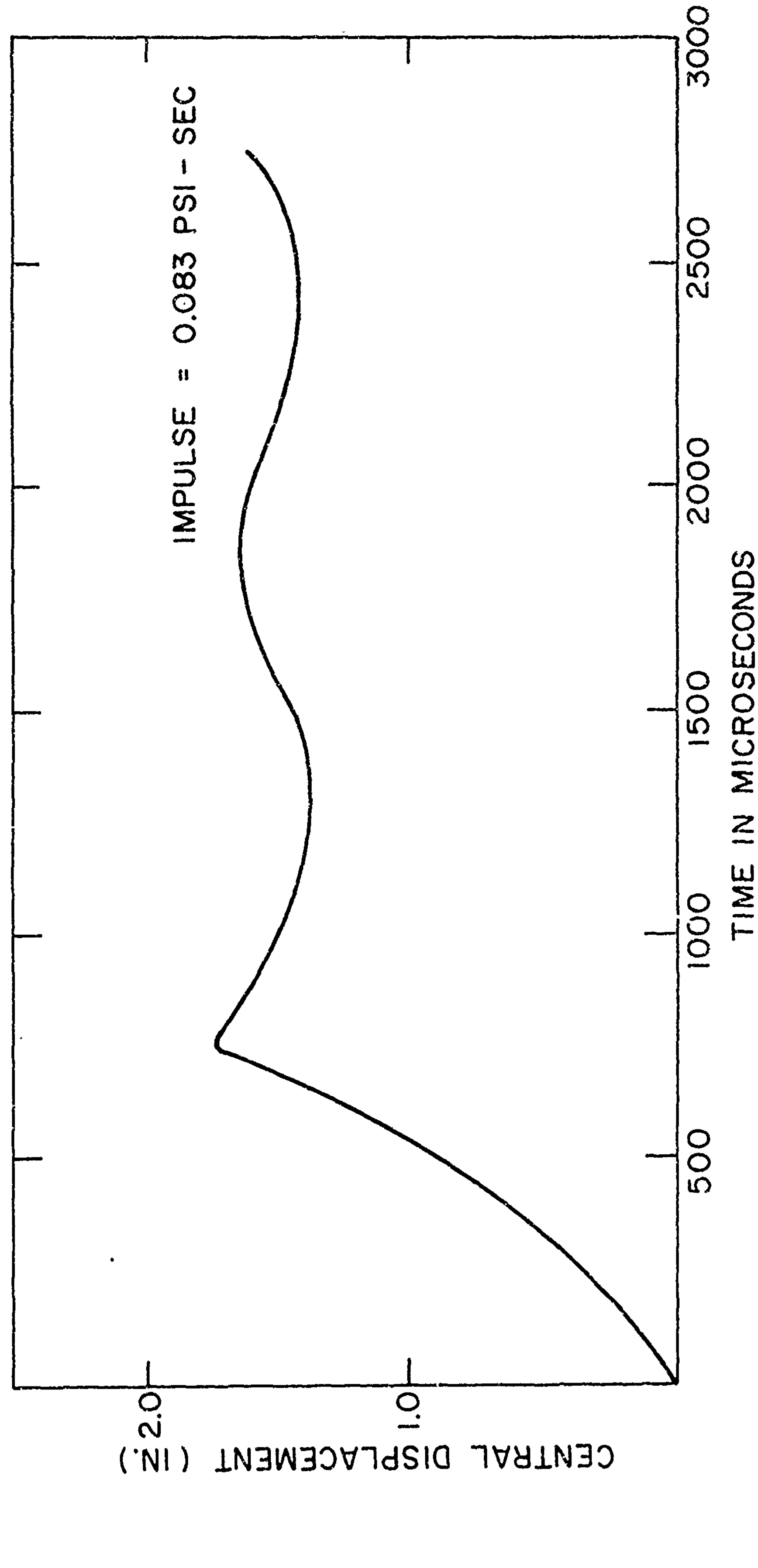

Fig. 2 Central Displacements of a Clamped Square Plate.

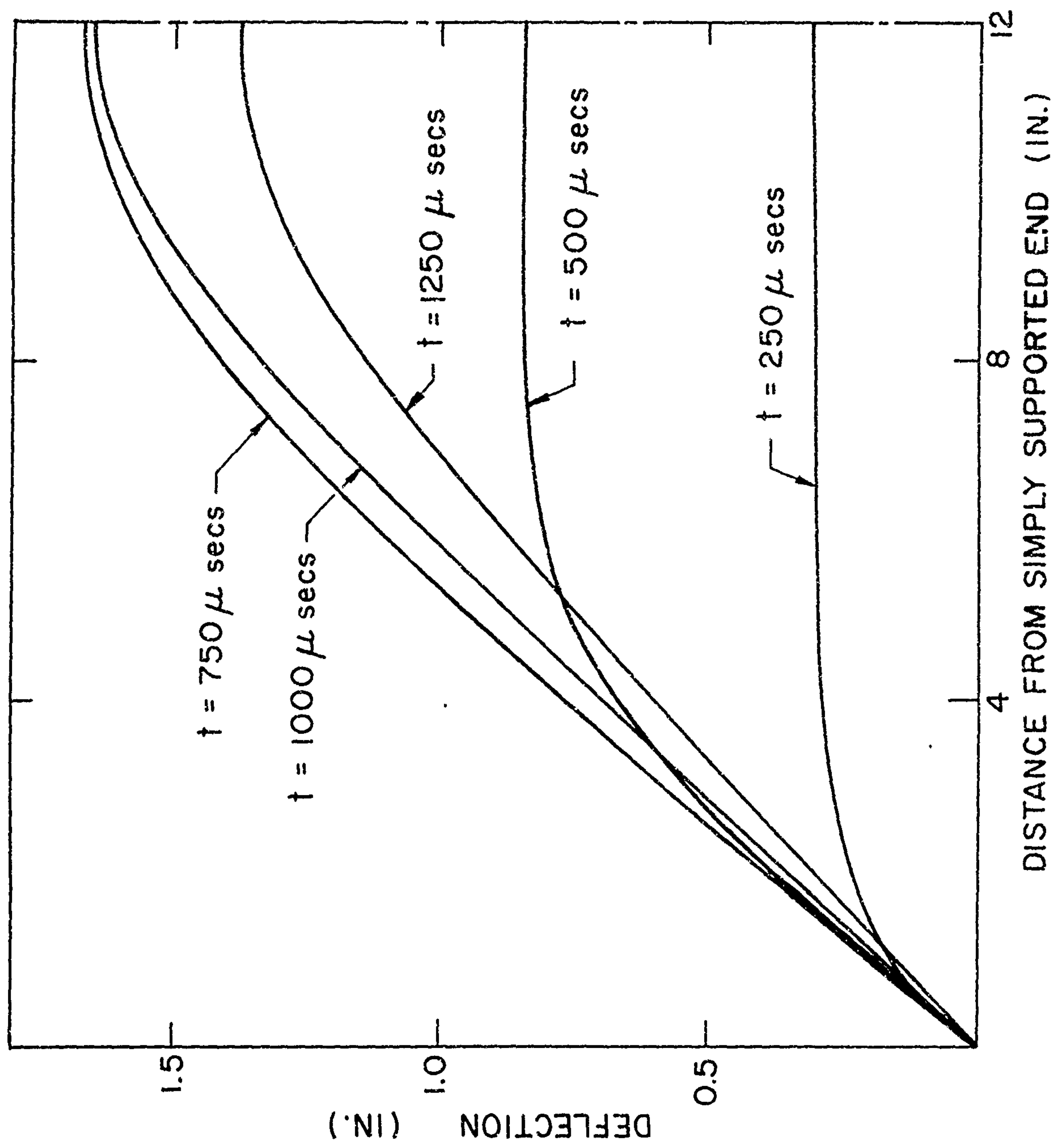

Fig. 3 Middle Span Deflections of a Simply Supported Square Plate.

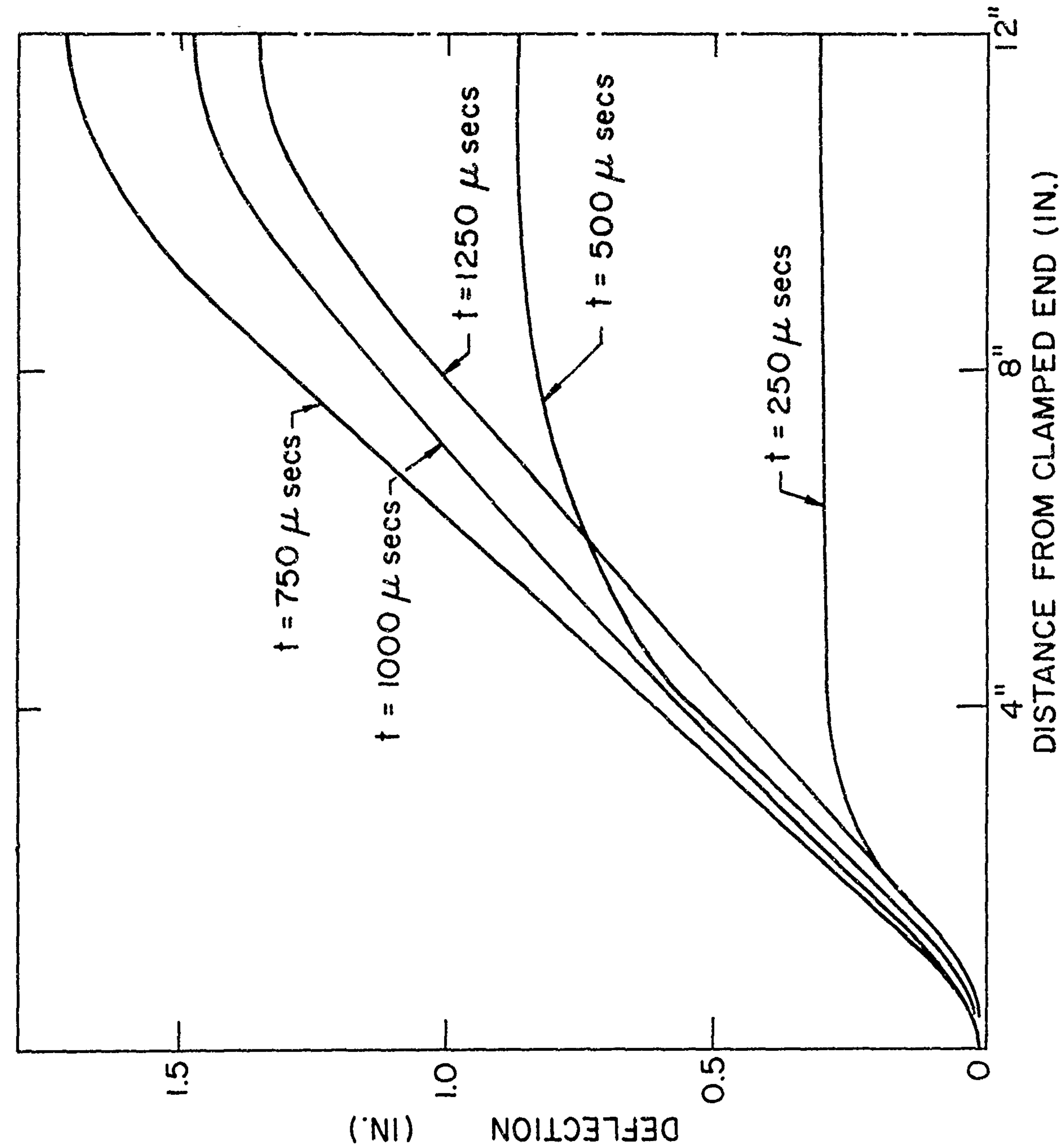

Fig. 4 Middle Span Deflections of a Clamped Square Plate.

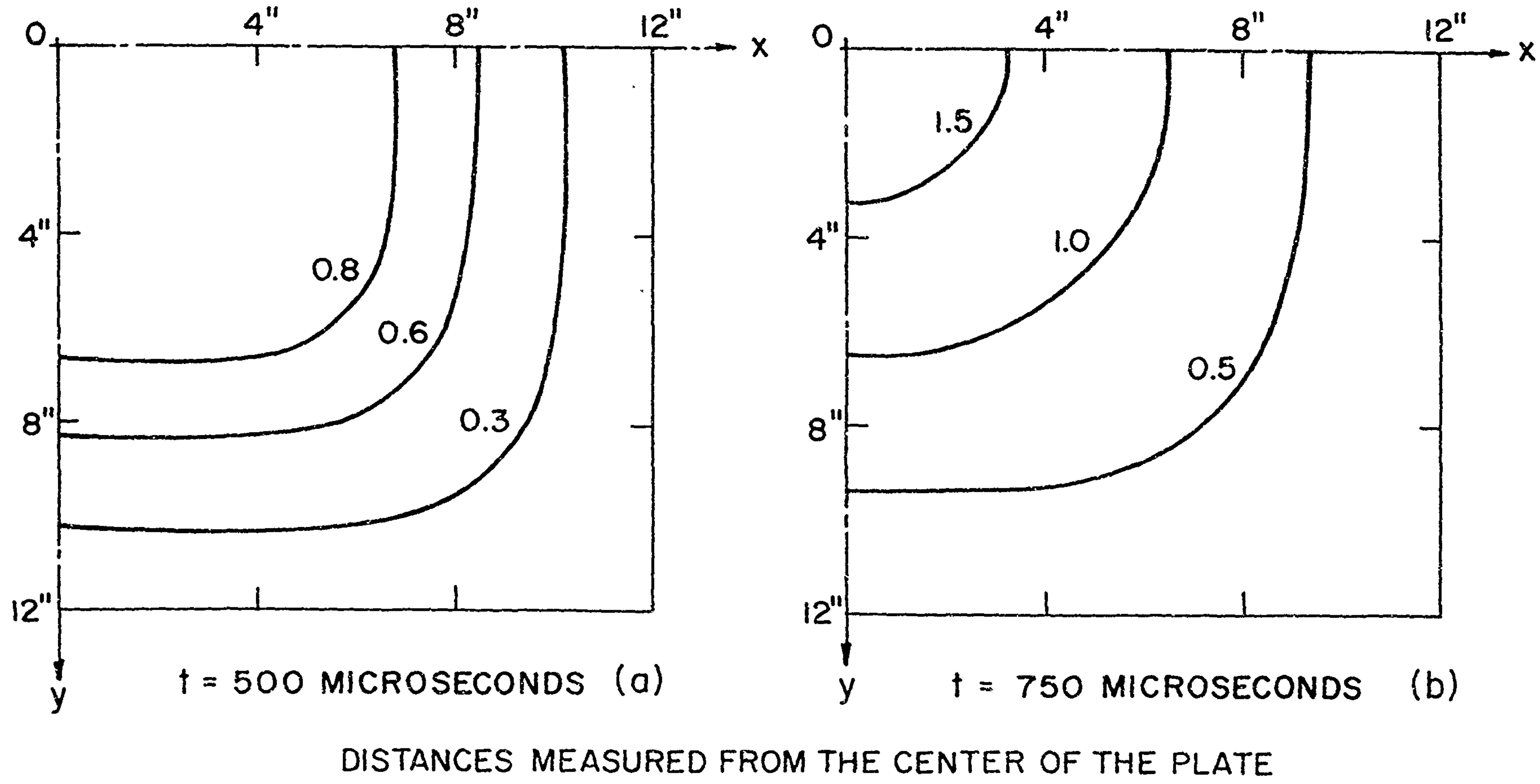

Fig. 5 Deflection Contours of a Square Plate.

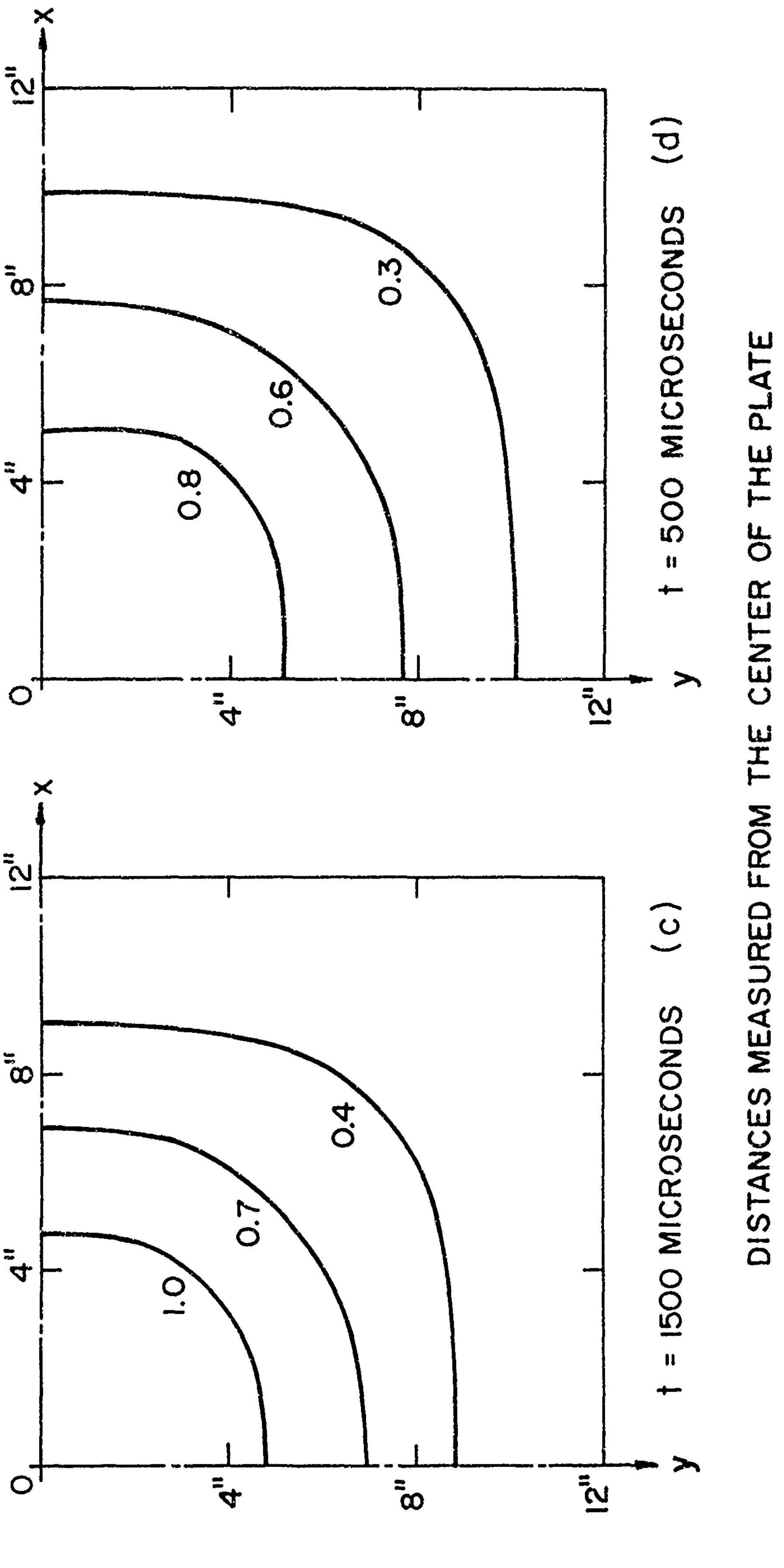

Fig. 6 Deflection Contours of a Square Plate.

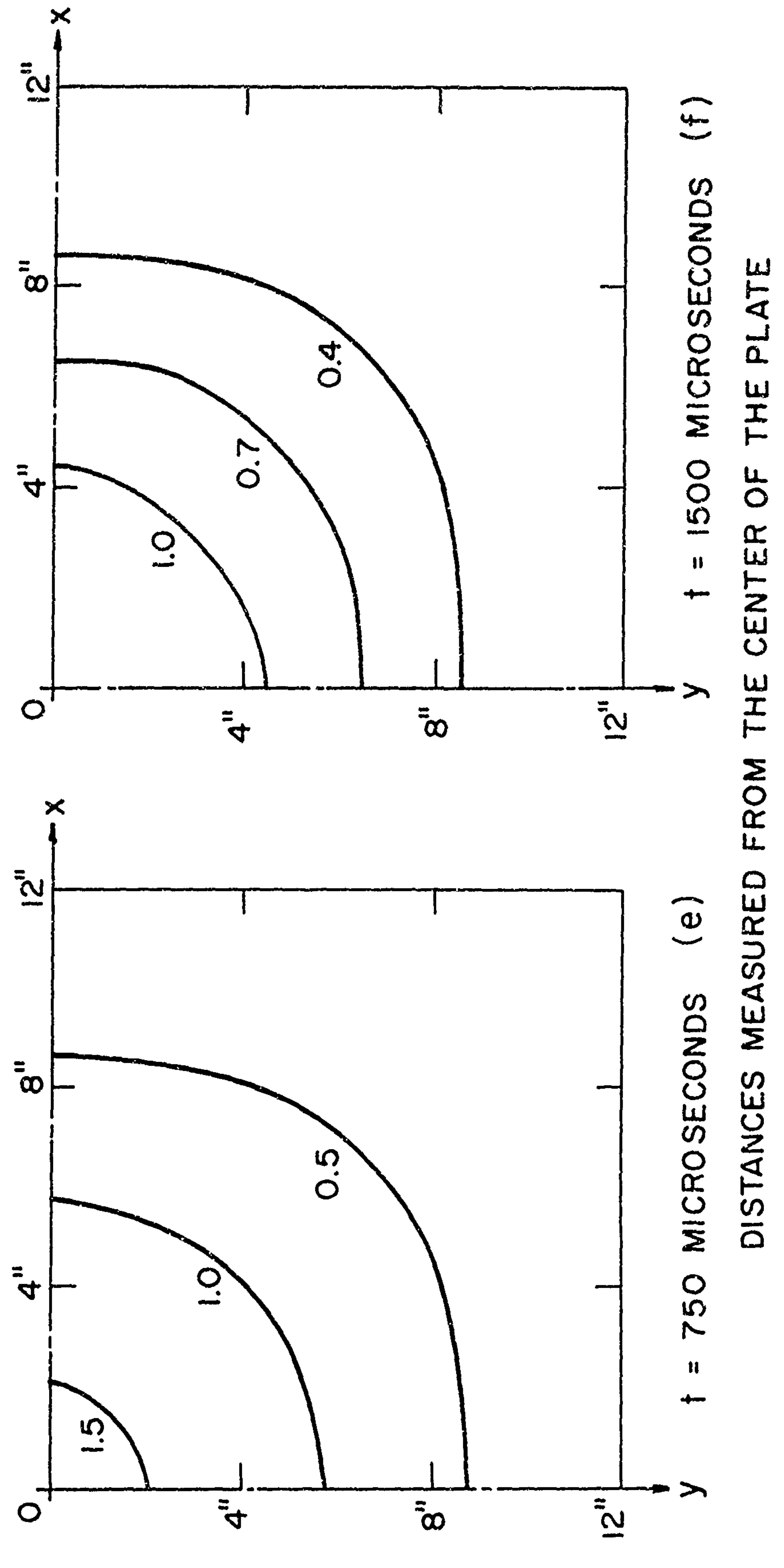

Fig. 7 Deflection Contours of a Square Plate.

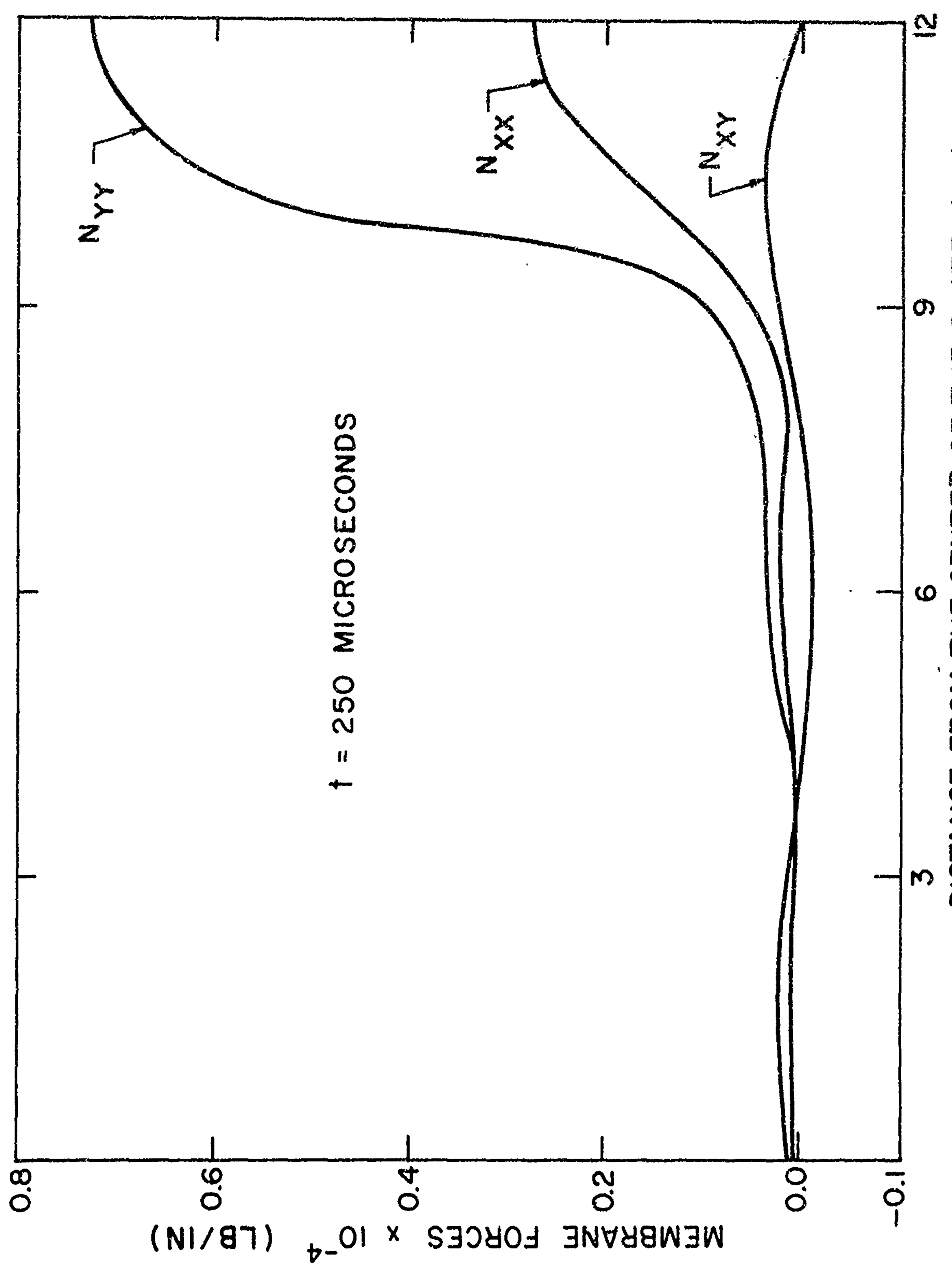

Fig. 8 Middle Span Membrane Forces of a Simply Supported Square Plate.

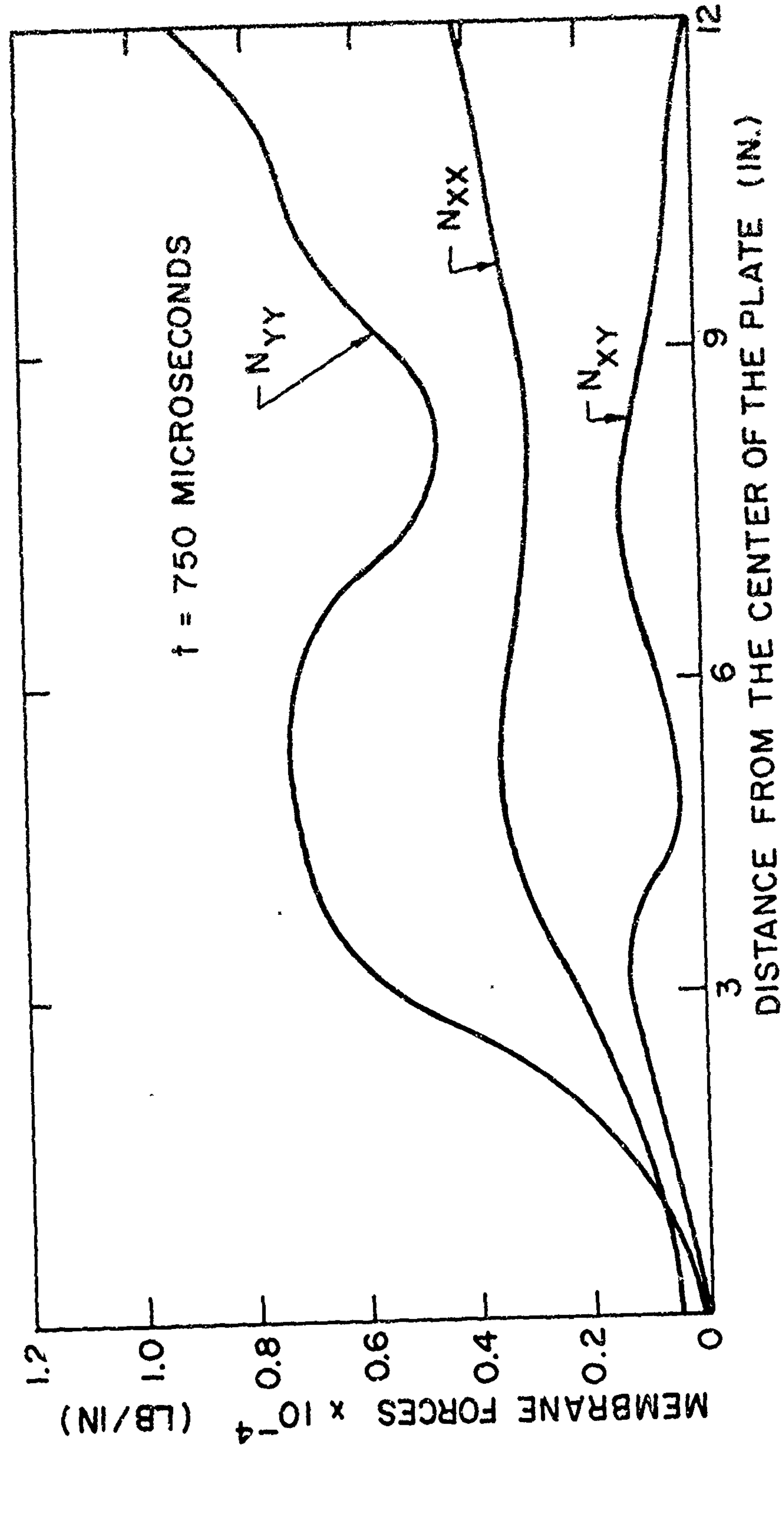

Fig. 9. Middle Span Membrane Forces of a Simply Supported Square Plate.

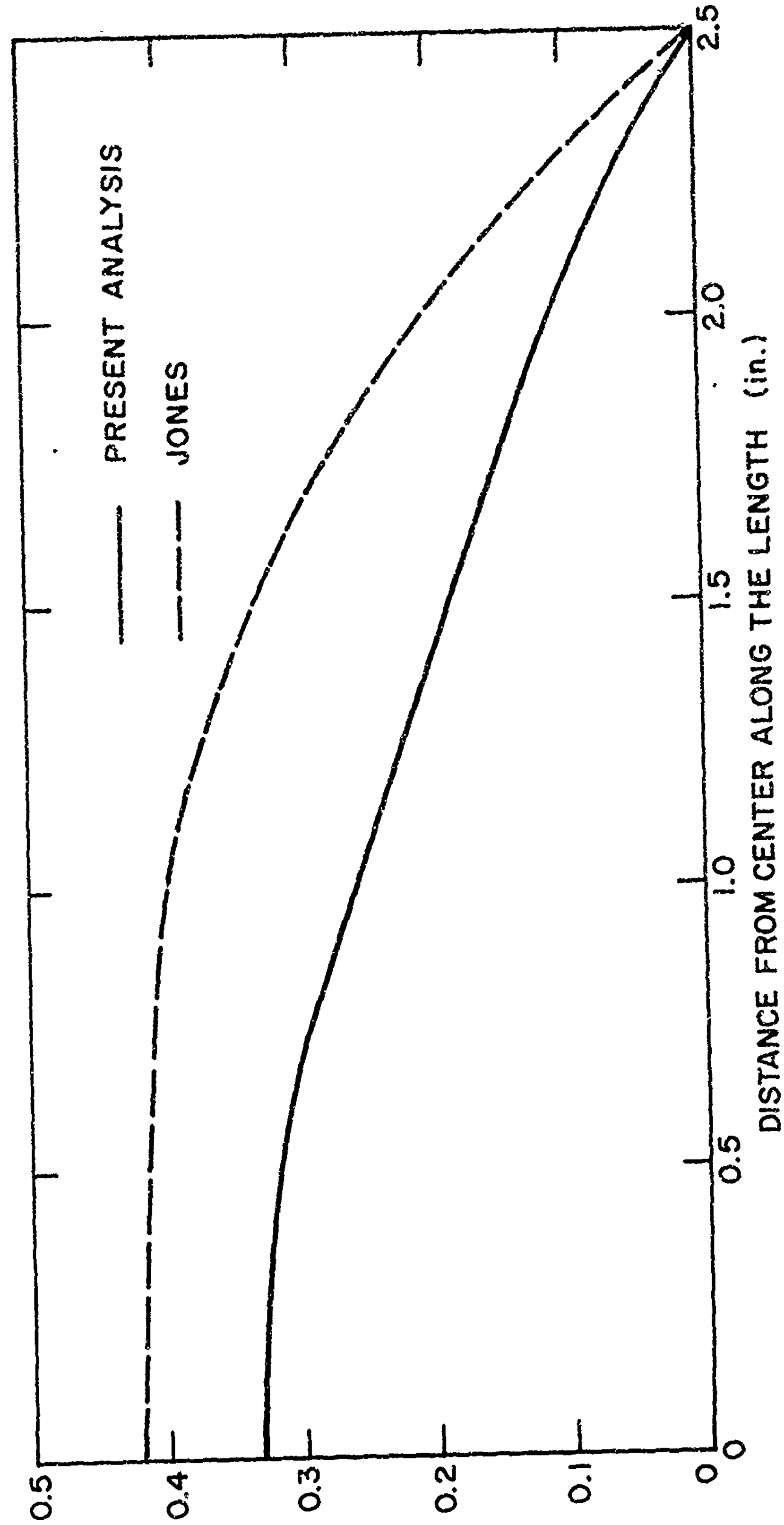

Fig. 10 Middle Span Permanent Profile of Aluminum 6061 T6 Plate.

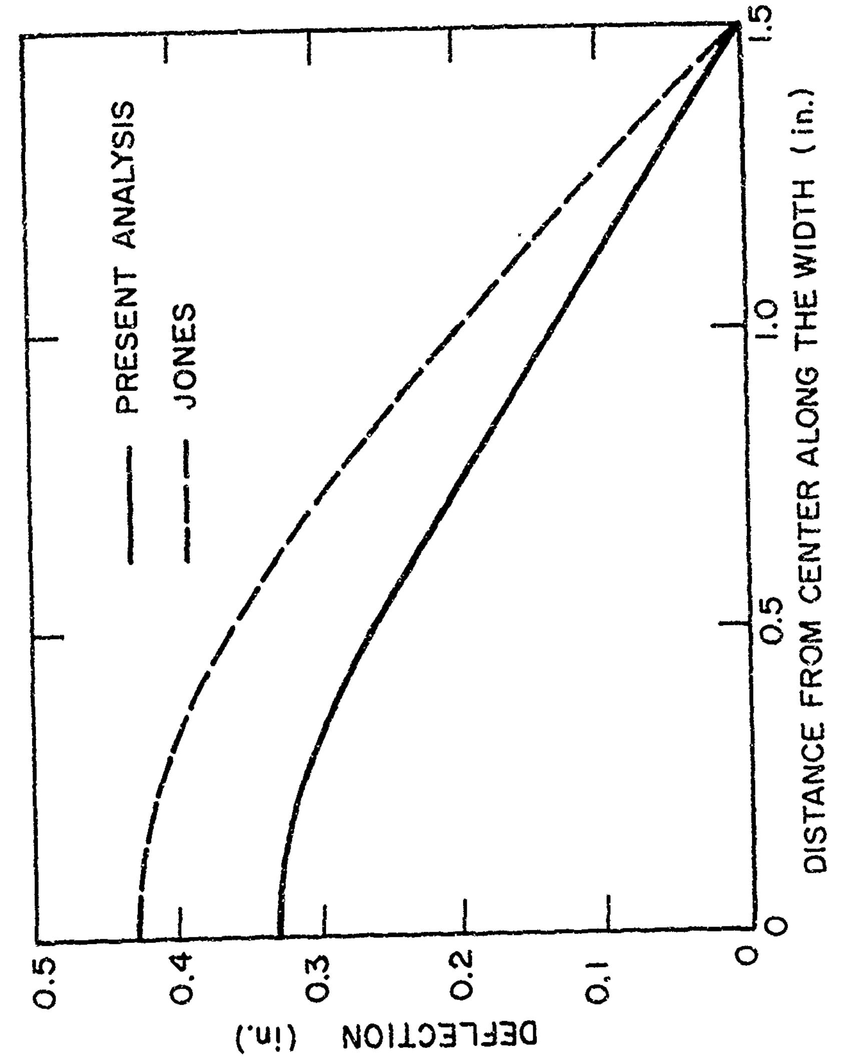

Fig. 11 Middle Span Permanent Profile of Aluminum 6061 T6 Plate.